A THIRD ORDER COMPUTER MODEL FOR STIRLING REFRIGERATORS

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ABSTRACT

A third order split-Stirling refrigerator model has been developed under an Independent Research program at Lockheed's Research and Development Division. The model consists of more than 90 nodes, with most of them inside the regenerator of the displacer where the temperature gradient is large. Conservation of mass, momentum and energy are solved at each node until the solution converges. The model is programmed using Continuous System Simulation Language (CSSL), which lends itself to solving a large number of complex differential equations simultaneously. Some special features of the computer program include: Smith's complex Nusselt number for heat transfer in the compressor and expansion space; transport in clearance gaps of the compressor and displacer pistons; Kays and London's correlations between heat and mass transfer in the regenerator; Amar and Cannon's pressure drop in the regenerator screens; Gorring's thermal conductivity of heterogeneous materials for the regenerator matrix; laminar and turbulent transport in the transfer line; and entrance/exit effects in all major contractions and expansions. Typical outputs of the model include temperature, pressure and gas flowrate throughout the system. Other outputs are P-V work at the compression and expansion space, and heat balance over the entire refrigerator unit. This model has been validated against experimental results of an Oxford type Lucas Stirling refrigerator with excellent agreement being found between the two.

INTRODUCTION

Analyses of Stirling refrigerators range in degrees of complexity from the back-of-the-envelope type calculations to complicated nodal network analysis. For a rough estimation, the refrigeration capacity (Q_R) of a refrigerator can be expressed as

$$\dot{Q}_R = C\,\omega T_E\,P_{MEAN}\,V_E / 2\pi \qquad (1)$$

where constant C approximately equals to $1x10^{-4}$ K^{-1}. The above equation is valid for large systems with temperature ranges from 80 to 120 K[1], and is known as the zeroth order analysis.

The classical analysis of the ideal Stirling cycle has been given by Schmidt[2] (first order approach). He assumes an isothermal process with no pressure drops in the system. A steady state process is also assumed together with a perfect regenerator. The basic Schmidt analysis is not too useful for designing refrigerators because it lacks information about the irreversible losses.

A second order analysis consists of the basic Schmidt cycle plus decoupled loss terms[3-5]. These loss terms include static (conductive) heat loss, shuttle loss, regenerator ineffectiveness, pressure drop, and pumping loss. There are a number of second order computer models in the public domain. Lucas Aerospace Co has recently developed a model called CMOD[6]. Some of its results will be published in Reference 7 together with the validation of the present model.

The computer program discussed in this paper belongs to a third order analysis. This approach was first pioneered by Finkelstein[8] in the mid 70's. It requires breaking the machine into a number of nodes. Majority of the nodes reside within the regenerator where the temperature gradient is large. Equation of continuity, momentum and energy (for both gas and solid)

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}\left(\frac{\dot{m}}{A_g}\right) \tag{2}$$

$$-\frac{\partial}{\partial t}\left(\frac{\dot{m}}{A_g}\right) = \frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\frac{1}{\rho}\left(\frac{\dot{m}}{A_g}\right)^2 + \left(\frac{A}{LA_g}\right)\left(\frac{\dot{m}}{A_g}\right)\frac{|\dot{m}|}{A_g}\frac{f}{2\rho} \tag{3}$$

$$\frac{\partial}{\partial t}(\rho u) = \frac{h_t A}{L A_g}(T_m - T) - \frac{\partial}{\partial x}\left(\frac{\dot{m} h}{A_g}\right) + \frac{\partial}{\partial x}\left(k_g \frac{\partial T}{\partial x}\right) \tag{4}$$

$$\rho_m c_m \frac{\partial T_m}{\partial t} = \left(\frac{\varepsilon}{1-\varepsilon}\right)\frac{h_t A}{L A_g}(T - T_m) + \frac{\partial}{\partial x}\left(k_m \frac{\partial T_m}{\partial x}\right) \tag{5}$$

are solved at each node until the solution converges. This method also requires equations of state, empirical fomulaes for heat transfer and friction factor which will be discussed in the following sections.

THE MODEL

Figure 1 shows the schematic diagram of the Lucas Stirling refrigerator. It consists mainly of a displacer and a compressor connected by a transfer line. Two linear motors are present, one in the compressor and the other in the expander housing. A molecule of gas at the compression chamber can either flow into the transfer line (towards the displacer), or it can penetrate the clearance gap and enters the compressor housing. Similarly, a molecule of gas at the displacer plenum (where the transfer line meets the expander) can either enter the

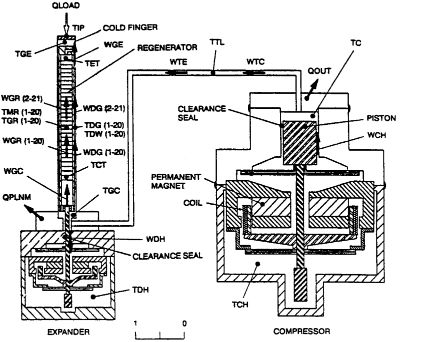


Figure 1 Lucas Stirling Cryocooler Schematic Diagram

displacer housing through the clearance gap, or it can flow into the expansion space via the regenerator or the displacer gap (between the regenerator and the coldfinger tube). Heat transfer takes place throughout the system by enthalpy transport. Heat transfer also occurs between the gas and regenerator matrix or the walls of the refrigerator. Figure 2 is a nodal diagram of the Lucas refrigerator. The majority of the nodes are located within the regenerator where the temperature gradient is large.

The Stirling Program of Refrigerator Model (SPRM)

The computer model developed under Lockheed's Independent Research program is written in Continuous System Simulation Language (CSSL). CSSL is an advanced language that can be used to solve large number of differential equations simultaneously. With four equation (Eq. 2 - 5) to be solved at each node and the number of nodes in the model, this language proves to be very useful.

Heat Transfer Coefficient in Compression and Expansion Space

Unlike most of the second order analysis, the present model does not assume isothermal or adiabatic compression. Heat transfer takes place between the gas and the wall according to the thermodynamic conditions of the system. Kornhauser and Smith[9] found that the heat transfer during compression and expansion is out of phase with the wall-bulk gas temperature difference, that the Newton's law of convection is inadequate in describing this phenomena. They express the heat transfer between the gas and the compressor (or expander wall) wall as a function of a complex Nusselt number.

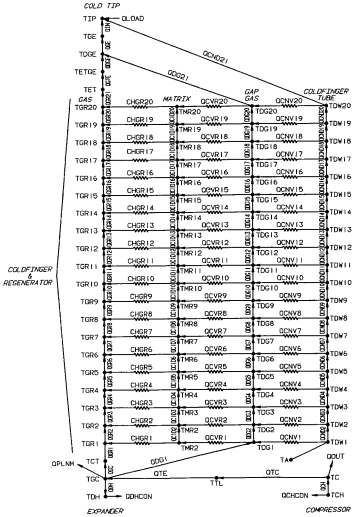


Figure 2 Nodal Diagram of Stirling Refrigerator Computer Model

$$\dot{Q} = \frac{k\,A}{D_h}\left[Nu_r\,(T - T_W) + \frac{Nu_i}{\omega}\frac{dT}{dt}\right] \tag{6}$$

With PeD_h/L_s larger than 100, the real and imaginary parts of Nusselt number are approximately equal and can be expressed by a power law

$$Nu_r = Nu_i = 0.98(Pe\,D_h\,/\,L_s)^{0.59} \tag{7}$$

For the Lucas Stirling refrigerator being modeled, 7 mm displacer and 10 mm compressor piston, $PeD_h/L_s >> 100$.

Heat Transfer and Pressure Drop in the Regenerator

The model makes use of Kays and London's experimental correlation[10] between heat and mass flow to calculate the heat transfer in the regenerator. Radebaugh[11] combined their results for various geometries in a single plot. For a given Reynold's number, the model calculates the Stanton number which in turn calculates the heat transfer coefficient.

As for the pressure drop, we elect to use the Amour and Cannon's expression[11] over that of Kays and London's. The main reason behind this is that Amour and Cannon's equation can correlate more type of screens (e.g. plain square, full twill, Fourdrinier, plain Dutch and twill Dutch). It also contains more characteristics of the screen (e.g. porosity, area to volume ratio, pore size etc.). According to Armour and Cannon, the friction factor across screens can be written as

$$f = \frac{\alpha}{N_{Re}} + \beta \tag{8}$$

where

$$f = \frac{\Delta P\, \varepsilon^2 D_p}{L \rho v^2}, \quad N_{Re} = \frac{\rho v}{\eta a^2 D_p}$$

Heat Conduction Along the Matrix of the Regenerator

The regenerator is randomly packed with a stack of screens where contact and even sintering of the matrix material is possible. Axial heat conduction along the regenerator matrix is thus very difficult to predict. Gorring and Churchill[12] have proposed a number of equations for thermal conductivity of heterogeneous material. For heat conduction through a square array of uniformly sized cylinders, they suggest the following empirical equation for the combined thermal conductivity,

$$k_x = k_g \frac{\chi - \varepsilon}{\chi + \varepsilon} \tag{9}$$

where

$$\chi = \frac{(1+\nu)}{(1-\nu)}, \quad \nu = \frac{k_m}{k_g}$$

for continuous media of the matrix material, the conductivity can be expressed as

$$k_x = k_m(1-\varepsilon) + k_g \varepsilon \tag{10}$$

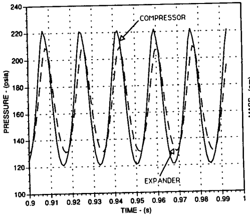
the actual thermal conductivity should be somewhere between Equation (9) and (10).

Fluid Dynamic within the Refrigerator

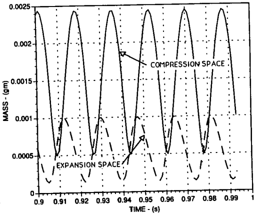
For fluid flow through annular gaps of the refrigerator (e.g. compressor and displacer piston clearance gap), the following equation is used

$$\dot{m} = \frac{S^3 D \pi}{12} \frac{\rho \Delta P}{\eta L} \tag{11}$$

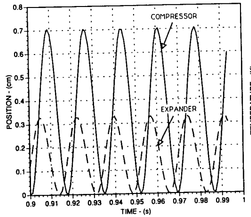
Frequency = 58 Hz
Radial Piston Clearance = 7.5 µm
Phase Angle = 80°
Compressor Housing = 320 K
Cold Tip Heat Load = 0.5 W



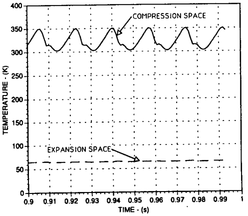
a. Compressor and Expander Piston Motion



b. Temperature Response in Compressor and Expander



c. Pressure Response in Compressor and Expander



d. Mass Flowrates in Compressor and Expander

Figure 3 Typical Stirling Cryocooler Program Output

According to this equation, the mass flow in the gaps is proportional to the cube of the gap size.

Throughout the rest of the system, we use Hagen-Poiseuille (laminar) or Blassius (turbulent) equations for tube flow. This process is highly iterative, as the Reynold's number which determines the flow regime is a function of the friction factor or the velocity. This is one of the most time-consuming processes in the computation, and possibly the main reason for the excessive run-time of this model. Entrance effects are also considered wherever appropriate.

RESULTS AND DISCUSSION

The model can be run on a VAX 8600, IBM 6000 workstation or CRAY. The output from the computer program consists of the following groups of results:

1) Temperatures, pressures, mass flow, and heat flow as a function of time and position.
2) Energy balance over the entire system, including various loss terms.
3) Integrated quantities such as P-V work at the compressor and displacer.
4) Performance parameters, e.g. coefficient of performance, efficiency etc.

Figure 3a shows the pistons positions as a function of time, the maximum stroke for the compressor and the displacer are 7 and 3 mm respectively. The difference between the amplitude of the two piston motions is the phase angle. Typical outputs of the program e.g. temperature, pressure and flowrates are given in Figure 3b to 3d.

CONCLUSIONS

The third order nodal network computer model described in this paper realistically simulates the behavior and the characteristics of a split-cycle Stirling refrigerator. It computes the performance of the machine without any fudge factors or scaling. Although a number of third order analysis are available in the literature, unfortunately most of them are neither well documented nor have they been validated by experimental results. The present model has been validated extensively by the experimental results of a Lucas built Stirling refrigerator[7]. Thus, it allows one to perform parametric studies to optimize the efficiency of the Stirling refrigerator by selecting various structural designs (dimensions and materials) and operating conditions (frequencies, phase angle, strokes etc) with great degree of confidence.

NOMENCLATURE

A	total area for heat transfer	u	internal energy
A_g	gas flow cross-sectional area	V	volume
a	area to volume ratio for screen	v	velocity
C	constant for Equation (1)	x	distance
c	specific heat capacity		
D	diameter		

D_h	hydraulic diameter	*Greek*	
D_p	pore size of screens		
f	friction factor	α	constant = 8.61
k	thermal conductivity	β	constant = 0.52
h	enthalpy	ε	porosity
h_t	heat transfer coefficient	η	viscosity
L	length	ρ	density
L_s	stroke length	ω	angular frequency
m	mass flowrate		
N_{Re}	Reynold's number for screen flow	*Subscript*	
Nu	Nusselt number		
P	pressure	E	expansion space
Pe	Peclet number	g	gas
Q_R	refrigeration capacity	i	imaginary
S	gap size	MEAN	mean value
T	temperature	m	matrix
t	time	r	real
		w	wall or matrix

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