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CHANNEL SIZE INFLUENCE ON THE HEAT FLUX DENSITY AT ZERO NET MASS FLOW IN THE NON-LINEAR TRANSPORT REGIME BETWEEN 1.2 K and 2.1 K

T.H.K. FREDERKING, S.W.K. YUAN, J.M. LEE , G.S. SUN *
University of California, BH 5405, Los Angeles 90024 CA (USA)

1. RESUME: On a mesuré le débit de séparateur de liquide-vapeur à temperature de l'helium II dans un regime non-lineaire. Un milieu poreux utilisé montre une influence de la taille des pores sur la perte de masse. On etend les études de transport de chaleur par convection dans les milieux poreux sans perte de masse. Les resultats experimentaux montrent l'influence des pores sur le flux de chaleur pour la permeabilité entre 10^{-8} cm^2 et 10^{-6} cm^2.

Les milieux poreux - le transfert de chaleur - He II - zero transfert de masse - regime non-lineaire.

2. INTRODUCTION

The technology of phase separators , e.g. Ref.1, and devices using thermomechanical forces, established by a temperature (T) gradient in He II, has been extended to smaller and smaller pore sizes of porous media. The work on phase separators has had the following results: 1. The normal fluid flow and related entropy and heat convection rates are controlling quantities in the relatively large pores of the IRAS type technology [1] ; 2. The convection rate of normal fluid in most data sets , reported in the literature up to this time, has been in the non-linear regime; 3. The usual "rate constant" (Gorter-Mellink constant), associated with non-linear flow, has been found to vary with the pore size. The constant appears to be proportional to the square root of the Darcy permeability (K_{Dn}).

As these results have been emerging in very recent time, the immediate question arising in this context is as follows: Does the entropy/heat convection rate at zero net mass flow (ZNMF) have the same type of pore size influence as in vapor liquid phase separation (VLPS) ? The present paper has the purpose of investigating this question for porous media and for fine ducts of simple shape at ZNMF. Basic equations[2] are discussed, and experimental data (Refs. 3 to 6) are compared to VLPS prediction. Further, conclusions are presented.

3. TRANSPORT MODES OF ZERO NET MASS FLOW

Using the two-fluid model, two limiting cases of linear and non-linear transport modes are considered in this section. The basic postulate is the density (ρ) condition for normal fluid (ρ_n) and for superfluid (ρ_s) :

$$\rho_n + \rho_s = \rho \qquad (1)$$

The mass flux densities (j) add up, $\vec{j} = \vec{j}_n + \vec{j}_s$, such that for ZNMF

$$\vec{j}_n = \rho_n \vec{v}_n = - \rho_s \vec{v}_s = - \vec{j}_s \qquad (2)$$

As a temperature gradient is imposed across the narrow duct system, the ideal He II thermostatics is characterized by London's equation

$$\text{grad } P = \rho \, S \, \text{grad } T \qquad (3)$$

(P pressure , S entropy per unit mass).

Linear Regime. The special thermomechanics of Eq.(3) is incorporated in the two-fluid equations describing superfluid and normal fluid motion. For negligible inertia, the change of the superfluid velocity (v_s) with time (t), in the absence of excitation of quantized vortex lines (Landau phase) is [2]

$$\partial \vec{v}_s / \partial t + \text{grad } \mu = 0 \qquad (4)$$

*) Visiting Scholar, PRC.

with a chemical potential increment, per unit mass, of $d\mu = - S\, dT + dP/\rho$. Eq. (4) may be rewritten as

$$\rho_s\, \partial \vec{v}_s/\partial t = - (\rho_s/\rho)\ \text{grad}\ P + \rho_s\ S\ \text{grad}\ T \qquad (5)$$

Because of a finite shear viscosity (η_n) of the normal fluid, an additional term is needed for the friction influence in the normal fluid equation:

$$\rho_n\, \partial \vec{v}_n/\partial t = - (\rho_n/\rho)\ \text{grad}\ P \ - \rho_s\ S\ \text{grad}\ T + \eta_n \nabla^2\ \vec{v}_n \qquad (6)$$

It is noted that the thermostatic case ($v_s \rightarrow 0$) in Equation (5) leads to

$$\text{grad}\ P = \text{grad}\ P_T = \rho\ S\ \text{grad}\ T \qquad (7)$$

(subscript T denoting thermomechanical pressure variation). Equations (5) and (6) are readily added, making use of Eqs. (1) and (7), (for $v_j \rightarrow 0$,)

$$0 = - \text{grad}\ P_T \ + \eta_n \nabla^2\ \vec{v}_n \qquad (8)$$

After insertion of Eq. (7) into Eq. (6), noting Eq. (1), one arrives at the following equation for the normal fluid motion

$$\rho_n\ \partial \vec{v}_n/\partial t = - \text{grad}\ P_T + \ \eta_n \nabla^2\ \vec{v}_n \qquad (9)$$

Equation (8) has been the basis for the Darcy law analog [3] for heat flow, at zero net mass flow, in porous media. Newtonian boundary conditions are imposed.

Non-Linear Regime. Heuristic arguments , relating to experimental evidence, and dimensional analysis results are utilized. Steady flow is considered. For retention of a certain symmetry, the substantial derivative (D./ Dt) of a lab-attached Euler frame is employed. This leads to replacement of $\rho_j \partial/\partial t$ on the left hand sides of Eqs. (5) and (6) by $\rho_s\ D\vec{v}_s/Dt$ and $\rho_n\ D\vec{v}_n/Dt$. Steady flow is assumed, and kinetic energies are added , subject to zero net mass flow of Eq.(2). By order of magnitude we have

$$\frac{1}{2}\ \nabla (\rho\ v_n^{\ 2}\ [\ \rho_n/\rho_s]\ )\ \sim \left\{\ \nabla P_T \ + \eta_n \nabla^2 \vec{v}_n \right\} \qquad (10)$$

From Equation (10) relevant dimensionless quantities may be extracted. The heat flow rate is expressed as normal fluid flow rate in dimensionless form:

$$Re_{v_n} = L_c\ \rho\ v_n\ /\ \eta_n \qquad (11)$$

The driving P-gradient may be obtained by considering the first two terms of Equation (10) : $|\text{grad}\ P_T|\ (\rho_s/\rho_n)\ /\ (\rho\ v_n^{\ 2})$. It is convenient to eliminate the velocity v_n by multiplication of the grad P_T - expression by $Re_{v_n}{}^2$. This results in a dimensionless driving term

$$N_{\nabla P_T}\ (\rho_s/\rho_n) = \ (\ \rho\ L_c^{\ 3}\ |\text{grad}\ P_T|/\ \eta_n^{\ 2})\ \ (\rho_s/\rho_n) \qquad (12)$$

Accordingly, one expects the following functional dependence : Re_{vn} is a function of the (dimensionless) number, Eq. (12) and of the pertinent ratios for the geometry considered. Guidance from ZNMF results for wide ducts suggests that

$$Re_{v_n}\ (\rho/\rho_s) = K_{GM}\ \ N_{\nabla P_T}^{\ 1/3} \qquad (13)$$

(Wide duct constant $K_{GM} \approx 10^1$, to first order, independent of T,P). Recent VLPS work has indicated a pore size-dependent constant. For ZNMF we rewrite the constant as $K_o^{\ *}$ (in contrast to VLPS). From VLPS results we expect $K_o^{\ *} = K_o\ (K_{Dn})$. The size $L_{cn} = K_{Dn}^{\ 1/2}$ is a measure of the throughput capability of the flow constriction during laminar flow at low speed.

4. COMPARISON OF ZNMF EXPERIMENTS WITH VLPS PREDICTION

In the non-linear regime considered, the characteristic length L_{cn} is chosen to represent the size of the flow passage. A generalized Darcy law analog [3] is used for the normal fluid ducts. The latter include fine slits [6] and the"single grain-single pore" system of Keesom et al.[4]. Further, a fibrous porous medium[5] is considered (K_{Dn} = 3.7 x 10^{-7} cm^2).

The exact slit geometry of Ref.4 has not been made available in the original publication. The duct walls are spherical surfaces with a large radius of curvature, compared to the slit width. Flow cross sections fortunately are available from a graphical presentation of heat flux densities (q) and tabulated power data. Figures 1 and 2 display data deduced from the slit information [4, 6].

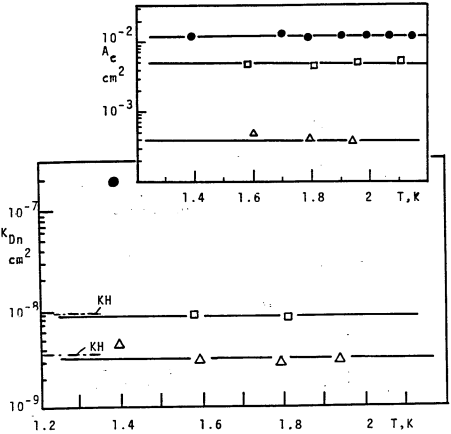

Fig. 1. Permeability K_{Dn} of Darcy convection of heat (entropy and normal fluid) through slits ; Ref. 4 : parameter, slit width (nominal values) : triangles 0.75 µm ; squares 9.3 µm ; circles 19 µm ; Ref. 6 : KH = -·-·- nominal K_{Dn} - values ;
Inset : cross sectional areas [4] ; data symbols as in Fig. 1 .

The inset of Fig. 1 shows the cross sections obtained versus the bath temperature. Figure 1 itself presents permeability data K_{Dn} (T) of three slits [4], evaluated from linear regime results. Included are nominal Darcy permeabilities ($d^2/12$) of annuli [6] of width d.

After determination of permeabilities, we obtain non-linear regime rates as K_o^* - results. The fibrous porous medium[5] is characterized by K_o^* = 7.5. Fig.2 presents a set of slit results using the K_o^* - value of 4.7 , Ref. 4. The heat flux density q is plotted versus the temperature difference (ΔT) across the slit for various bath temperatures. The inset of Fig.2 displays K_o^* as a func-

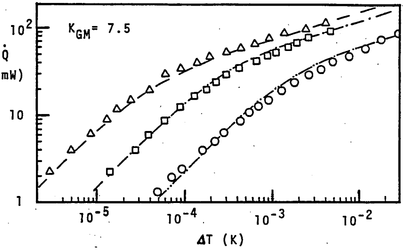

Fig. 3 . Heater power $\dot{Q}$ versus temperature difference for a fibrous porous medium; bath temperature : – – △ 1.512 K ; –·–·· □ 1.756 K ; ○–··– ○ 2.018 K .

Figure 4 presents the various rate constants K_o^* deduced for slits and porous plugs . For porous VLPS plugs the following equation has been proposed:

$$K_{VLPS} = [\, K_{GM,\infty}^{-2} + (10^{-7}/\, K_{D_n})\,]^{1/2} \qquad (15)$$

$K_{GM,\infty}$ is the wide duct reference value 11.3. All of the present ZNMF data for slits are above Eq. (15). The order of magnitude discrepancy corresponds to Newtonian fluid behavior : Fine ducts with smooth walls have a higher throughput than porous media(for a specified K_{Dn}) . Fig.4 includes a sintered porous plug result in reasonable agreement with Eq. (15).

5. CONCLUSIONS

In summary, it is seen that narrow ZNMF passages with rather smooth walls are not in quantitative agreement with porous plug VLPS results. The difference appears to be in the order of magnitude range expected from hydrodynamics [7] . Nevertheless , a duct size dependence is indicated by the data. The few porous media data available for ZNMF appear to be in good agreement with the VLPS plugs . It is concluded that ZNMF plug data are similar to VLPS plug results.- Further details are given in Ref. 8.

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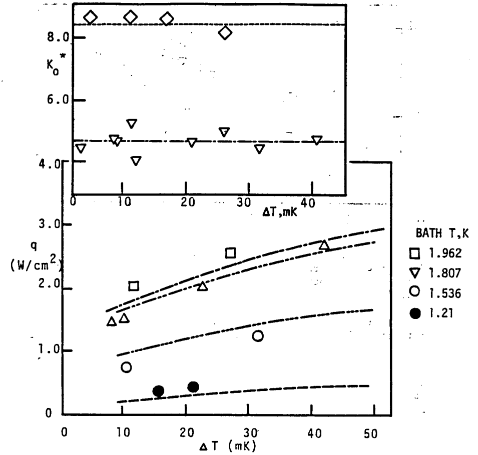

Fig.2. Heat flux density of a 9.3 µm slit [4] versus temperature difference (K_o^*= 4.7) ; Inset : K_o^* versus ΔT : ◇ slit width 19 µm ; ▽ slit width 9.3 µm .

tion of T-difference. To first order, the particular K_o^* - value indeed characterizes non-linear transport .

For the fibrous porous medium [5], exhibiting a gradual transition from linear flow, with $q=q_{oL}$, to non-linear flow , with $q = q_{oNL}$, an interpolation equation is useful :

$$q_{ZNMF} = q_o = (q_{oL}^{\ 3} + q_{oNL}^{\ 3})^{1/3} \qquad (14)$$

The subscript zero denotes the superficial heat flux density (=power divided by the total porous plug cross section). The case of small ΔT is the asymptote of linear Darcy convection (thermomechanical Darcy law analog). The normal fluid speed is $v_{no} = K_{Dn} |\mathrm{grad}\ P_T| / \eta_n$; grad P_T of Eq.(3) ; $q_o = \rho\ S\ T\ v_{no}$. At large ΔT, the q_o - values of the 1/3 power law regime , Eq. (13), are reached. Fig.3 shows good agreement of data with Eq.(14) (within data scatter). It is expected that similar plugs with a <u>finite</u> pore size spectrum will be described by Eq.14.

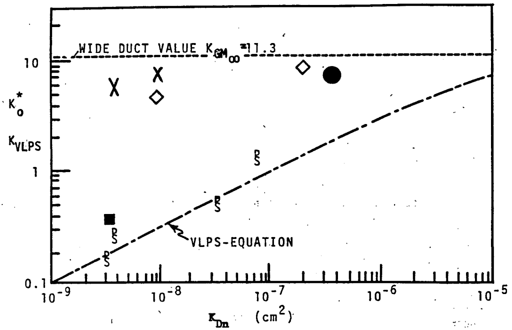

Fig. 4. Comparison of rate constants for ZNMF (K_o^*) with phase separator constants (K_{VLPS}) and the VLPS Equation (15) ; § VLPS data [8] ; full symbols : ZNMF porous media ● fibrous medium [5] , ■ stainless steel plug (nominal filtration rating 2 µm) ; X ◇ ZNMF slit data [4 6].

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