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Equations for Heat and Mass Flow of Non-Newtonian Fluid Through Porous Media: Liquid Helium II—Helium-4 Vapor Separation

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ABSTRACT

In novel phase separator systems well-defined venting of vapor from liquid is accomplished by means of porous plugs using the fountain effect as liquid-retaining phenomenon. A small finite mass flow is superposed on the heat transport rate associated with zero net mass flow. The present work focuses on two theoretical approaches proposed as two-fluid transport in one case and semi-classical convection in the other case. Data obtained up to this time have shown partial agreement with the well known He II two-fluid description. However in the data range covered by present and other experiments the difference between the two heat transport rate predictions is small compared to porous media tolerances and data resolution.

NOMENCLATURE

A_{tot} total cross sectional area of porous plug
 c_p specific heat at constant pressure
 H enthalpy
 j mass flux density (j_j ; $j = n, s$)
 K_{GM} Gorter-Mellink constant, dimensionless
 K_p permeability (K_{pj} , $j = D, n$)
 K_v flow parameter
 L length of porous plug, (L_D duct length)
 \dot{m} mass flow rate
 P pressure (P_p saturated vapor pressure)
 q heat flux density
 s entropy per unit mass
 t time
 T temperature

v velocity (v_j , $j = n, s$)
 w relative velocity ($v_n - v_s$)
 z position coordinate

η_n shear viscosity of normal fluid
 λ latent heat of vaporization
 μ chemical potential
 ρ density
 σ surface tension

Subscripts

d downstream ; D Darcy value
 GM Gorter-Mellink
 n normal fluid value
 o superficial value, e.g. $j_o = \dot{m}/A_{tot}$
 s superfluid
 u upstream
 $ZMFP$ zero net mass flow
 λ lambda transition

INTRODUCTION

Large scale cryogenic systems, operated in recent time near 2 K, include utilization of porous media, mostly sintered porous plugs. The plugs are essential components in novel phase separation devices for liquid Helium-4 vessels which contain the superfluid phase He II below the lambda temperature. For $T < T_\lambda$ the following requirements are to be met:

Reliable passage of mass through the plug for the heat input imposed on the liquid bath; Stable flow of entropy toward the downstream location; Stable vapor-liquid phase separation preferably near the downstream side of the plug; Establishment of a well-defined fountain effect pressure gradient preferably close to thermostatic conditions in the pores of the

plug.

Several groups have provided experimental data for the selection of suitable plug conditions and sizes. In theoretical attempts covering the finite mass flow rates in the porous media, two modelling efforts have been proposed: The first most popular theory is the two-fluid model of Landau and Tisza and London (1), (2), (3). The second transport description is a semi-classical model proposed by Van Sciver (4). In view of data uncertainty, and the discrepancies in the theory, the present work has the purpose of investigating details of the theoretical prediction of the heat transport rates. In addition, one particular plug has been studied in experiments aiming at a better resolution of the porous plug separators. In the next section we describe consequences of the two-fluid model to be deduced for the linear regime of transport at low energy expenditure. Subsequently the semi-classical model is rewritten in terms of the two-fluid quantities of the preceding section with emphasis on departures from the zero net mass flow case (pure heat transfer by counter-convection). Finally, the experimental studies are outlined and conclusions are drawn.

TWO-FLUID MODEL FOR PHASE SEPARATORS

The basic statements for the non-Newtonian fluid system are listed as follows:

Postulate for the densities of the two fluids:

$$\rho = \rho_s + \rho_n \quad (1)$$

Mass flux density postulate:

$$\vec{j} = \rho \vec{v} = \rho_s \vec{v}_s + \rho_n \vec{v}_n = \vec{j}_s + \vec{j}_n \quad (2)$$

Relative velocity

$$\vec{w} = \vec{v}_n - \vec{v}_s \quad (3)$$

Heat flux density

$$\vec{q} = \rho_s \vec{w} S T \quad (4)$$

The classical, simplified first law statement for the interfacial domain of vapor and liquid on the downstream side is expressed as

$$\vec{q} = \vec{j} \lambda \quad (5)$$

The configuration of interest is shown schematically in Figure 1. It is noted that the difference $\Delta T = T_u - T_d$ generates the fountain pressure gradient

$$\text{grad } P = \rho S \text{ grad } T \quad (6)$$

The ΔT imposed produces a diminished concentration (ρ_s/ρ) in the vessel with respect to the downstream location. This concentration difference generates flow of superfluid toward the warmer side of the plug. The second law of thermodynamics predicts heat transport from the upstream to the downstream side. Finally, according to Equation (5), there is a finite mass flow in the axial direction through the plug parallel to normal fluid and heat flow.

The directional properties of the two-fluid system may be recognized by relating the heat transport rate \vec{q} to fluid transport, e.g. \vec{v}_s . The normal fluid velocity v_n may be eliminated from Equation (2) using Eqs. (3) and (5)

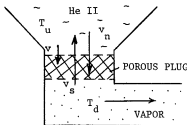


Figure 1. Schematic diagram of porous plug operation in He II in the phase separation regime.

$$\vec{j} = \vec{v}_s \rho + \vec{w} \rho_n = \vec{q} / \lambda \quad (7)$$

The relative velocity w may be eliminated from Equation (7) by means of Equation (4):

$$\vec{q} = -\vec{v}_s \rho_s S T \left\{ (\rho_n/\rho) - (\rho_s/\rho) (ST/\lambda) \right\}^{-1} \quad (8)$$

At thermodynamic equilibrium (ρ_s/ρ) is a unique function of T for bulk liquid. This allows evaluation of the sign on the right hand side of Eq. (8). It turns out that $\vec{q}/(-\vec{v}_s)$ is always positive in the T -range of operations covered in the present work. Thus, q is in counterflow to v_s as sketched in Figure 1. A special case is zero net mass flow with $j = 0$ and $q_{ZMNF} = \rho v_n S T$.

Using a similar procedure for the evaluation of q as a function of v_n , we eliminate v_s from Equation (2) by means of Equation (3). Finally, w is expressed in terms of the quantities of Equation (4). The result is written as

$$\vec{q} = +\vec{v}_n \rho S T (1 + ST/\lambda)^{-1} \\ = q_{ZMNF} / (1 + ST/\lambda) \quad (9)$$

As the temperature is reduced sufficiently, with $ST \ll \lambda$, we obtain close agreement of the q -values of vapor-liquid phase separation with the zero net mass flow case.

There is a tortuous path of fluid through the pores of the plug. Figure 2 depicts theoretical possibilities. Figure 2 a shows different fluid flow directions of the various vectors, and Figure 2 b presents parallel velocities for vapor-liquid phase separation.

SEMI-CLASSICAL CONVECTION MODEL

Zero net mass flow corresponds to "free convection" in a fluid whose motion is frequently restrained by walls sufficiently far away from a heated object. There is a linear regime and a turbulent regime of counterflow of v_s and v_n (Gorter-Mellink flow). However, once a finite mass flow rate is switched on, one may encounter semi-classical convection behavior with

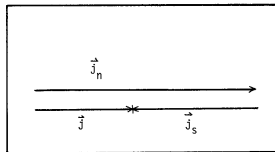
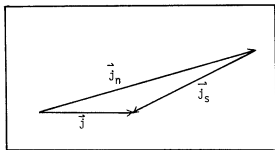


Figure 2. Examples of vectors of the two-fluid model description of vapor-liquid phase separation;
 a. Different directions of superfluid velocity, normal fluid velocity and mass flow velocity;
 b. Parallel velocities .

a finite velocity (v) superposed on the Gorter-Mellink counterflow. Van Sciver (4) has presented this case of convection adopting an Euler frame for the macroscopic description of the fluid motion. Thus, the usual thermal energy equation of a classical system is replaced by

$$\rho \frac{dH}{dt} \equiv \rho \left\{ \left(\frac{\partial H}{\partial t} + v \left(\frac{\partial H}{\partial z} \right) \right) \right\} = \partial q_{GM} / \partial z \quad (10)$$

For steady flow we have $\partial H / \partial t = 0$. Presuming this mode, and quasi-steady motion respectively, we simplify Equation (10), arriving at

$$\rho v \left(\frac{dH}{dz} \right) = d |q_{GM}| / dz \quad (11)$$

The geometry of Reference (4) is shown in Figure 3. Flow takes place in a duct with adiabatic walls, and a heater provides thermal energy supply at the center ($z = 0$). A finite flow rate is generated externally by a suitable pump system. The boundary conditions are:

$$z = 0 : T = T_u \quad (12)$$

$$z = \pm L : T = T_d = T_u - \Delta T \quad (13)$$

A global condition for steady flow is obtained for the domain $0 \leq z \leq L$:

$$q \Big|_{z=0} = q_{GM} + \rho c_p \Delta T v \quad (14)$$

After integration of Equation (11), and elimination of the integration constant, Equation (14) predicts the effect of v on the heat transport capability of the duct system. As v becomes negligible, the usual Gorter-Mellink heat transport is recovered, and for

high mass flow rates (ρv), (high thermal energy convection rates ($\rho v c_p \Delta T$)), Equation (14) predicts a negligible contribution of q_{GM} .

In contrast to the preceding two-fluid model, the semi-classical approach predicts an enhancement of the heat transport rate through the plug. Consistency with Van Sciver's model is incorporated in the two-fluid equation by replacing Equation (8) by

$$\vec{q} = - \vec{v}_s \rho_s ST \left\{ \left(\rho_n / \rho \right) + \left(\rho_s / \rho \right) \left(ST / \lambda \right) \right\}^{-1} \quad (15)$$

The difference between the two functions is small, in particular at low T as the separator operation approaches zero net mass flow. Figure 4 shows the thermophysical property functions in Equations (8) and (15).

In Reference (4) a flow parameter has been used which is converted readily to the usual two-fluid quantities for He II (5), (6). According to Equation (14), the ratio of q / q_{GM} is a simple function of ($\rho c_p \Delta T$) to q_{GM} for the length L ; (subscript $z = 0$ omitted). A quantity K_v , proportional to the latter ratio, has been convenient for a comparison with experimental data (7). K_v contains mutual friction information related to the interaction of vortices with normal fluid (8). Using the present two fluid variables, we rewrite K_v of Reference (4) as

$$K_v = v \left(2 / K_{GM} \right) \left\{ \frac{c_p}{S T} \right\} \left(\rho / \rho_s \right)^{4/3} \left[\left(\Delta T \right)^2 L / \beta \right]^{1/3} \left(\rho_n / \eta_n \right)^{1/3} \quad (16)$$

($K_{GM} = 11.3$ in bulk He II (5)).

The theory of Reference (4) including a comparison with the data of Johnson and Jones (7) is shown as q / q_{GM} versus K_v . (For details we refer to Ref. 4). In the semi-classical model, parallel flow of normal fluid and mass is accompanied by an enhanced heat flow rate. In contrast, the two-fluid model predicts a reduction in the heat transport ratio, for the separator case, as a finite v is imposed. The counterflow of v and q_{GM} is a reduction of the ratio q / q_{GM} also for the semi-classical model.

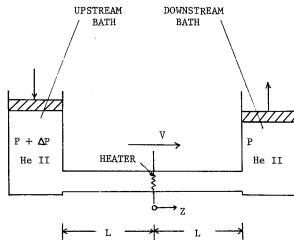


Figure 3. Insulated duct geometry of the semi-classical model, (modified system).

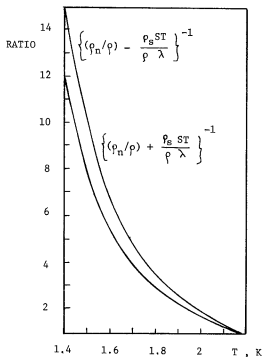


Figure 4. Thermophysical properties associated with the flow rate ratios of the two-fluid model and the semi-classical convection model

The inset in Figure 5 displays the velocity v as a function of the flow parameter K_v for constant thermophysical properties, i.e. $\Delta T \ll T$. The order of magnitude of the velocity in the present experiments appears to be near the order of K_v of 0.01. In this case, the difference between the two theoretical ratios (q/q_{CM}) is small compared to experimental resolution.

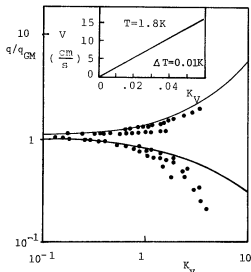


Figure 5. Heat transport ratio versus flow parameter (4);

Inset: Velocity of semi-classical model as a function of the flow parameter; ($L = 1$ cm).

EXPERIMENTS

The experiments are a continuation of previous phase separator work with a rotating shutter system (9). The motor-shutter assembly has been eliminated while retaining the essential components described in Reference (9). In the present runs a bronze plug (Pacific Sintered Metals, nominal particle retention rate of filtration in the 5 to 15 μ m range) has been used. The apparatus is shown schematically in Figure 6. Danger of pump oil backflow has been eliminated by a liquid nitrogen trap. The porous sintered plug is located in the interior section of the cryostat at the bottom of a central vapor vent tube. A liquid jacket around the vent tube represents the liquid He II vessel. The entire low temperature assembly is located in a He II bath and instrumented (9). The previous differential pressure

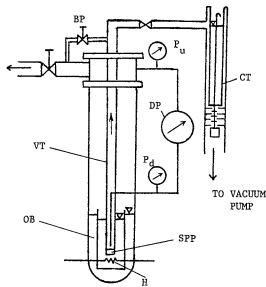


Figure 6. Experimental system for vapor-liquid phase separation (schematically);

- CT Cold trap;
- H Heater
- OB Outer bath
- SPP Sintered porous plug;
- VT Vent Tube carrying He^4 vapor;
- DP Differential pressure transducer for ΔP_v determination
- BP Bypass valve

transducer (Validyne DP 15) has been replaced by a more sensitive unit (Validyne DP 18).

A sample of data sets is contained in Figure 7. Figure 7a shows the liquid level position versus the time t . Figure 7b presents the superficial mass flow velocity $v_o = (\dot{m}/\rho)/A_{c,PE}$ versus t , as deduced from Figure 7a. Figure 7c depicts the temperature difference $\Delta T = T_u - T_d$, measured by carbon resistance thermometers.

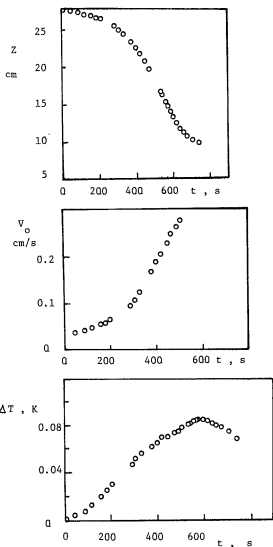


Figure 7. Data sets obtained with sintered bronze plug; a. Liquid level versus time; b. Superficial velocity v_0 versus t ; c. temperature difference ΔT versus t .

The previous transient technique (9) has been modified to include slow changes of state near quasi-steady operation. Both cases, $dT_c < 0$ and $dT_c > 0$ have been investigated. A temperature drift results readily from a mismatch in mass flow capability, for a particular valve setting, and heat supply to the vessel, and venting needs respectively. Heat capacity corrections have been applied to the data to obtain the steady state results.

DATA DISCUSSION AND CONCLUSIONS

The prime quantity characterizing the throughput of a porous plug is the Darcy permeability K_{PD} . The room temperature value has been obtained as $7.36 \times 10^{-8} \text{ cm}^2$. At low temperatures the value of the normal fluid throughput may be attained when q/q_{GM} approaches unity, provided the flow undergoes a transition to the linear regime. At low T , the departure of q from the value q_{ZMFE} is quite small,

as discussed in conjunction with the two-fluid prediction of phase separator phenomena. Table I lists K_{pn} -values deduced from the runs which lead to an upper bound if the low- T , linear regime cannot be attained due to pumping system limits.

TABLE I. Effective normal fluid permeabilities of the non-linear regime of transport during vapor-liquid phase separation

RUN	$10^9 K_{pn}$	\bar{T} , K
1	6.06	1.72
2	6.25	1.66
3	9.26	1.59
4	11.0	1.53
5	22.3	1.41

Early porous media data of phase separation runs have been inspected on the basis of dimensionless numbers deduced from the two-fluid model (10). Data trends of these initial exploratory runs appeared to be in part contradictory. Because of this unsatisfactory behavior, the Klipping group (11) has conducted extensive investigations of a better defined geometry incorporating a slit system. In recent time actual flight performance data for the porous plug of the IRAS system have become available (12). It appears that several results are quite consistent as soon as two-fluid concepts are applied, and the present data appear to be in reasonable agreement with data trends in view of porous media tolerances.

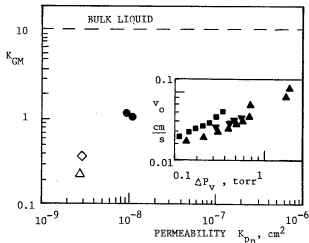


Figure 8. Gorter-Mellink constant K_{GM} (dimensionless) versus permeability of porous plugs; Basis: superficial value v_0 ; Inset: Mass flow velocity v_0 versus vapor pressure difference; ● Present data; ◇ Result for $j = 0$ (zero net mass flow) of Reference (13); △ Phase separator data of Reference 13 for sintered stainless steel plug (nominal particle retention of $2 \mu\text{m}$).

It is noted from Figure 8 that the phase separator plugs considered have Gorter-Mellink constants one order below the bulk value, or even lower values. In addition, we mention that forced flow results for a wide tube (14) did not show a discernible departure of data from the Gorter-Mellink prediction up to velocities of the order of 0.1 cm/s .

The following conclusions are drawn:

1. In the range of porous plug operation covered in the present phase separation runs, the difference between the q-ratio prediction of the two-fluid model and the semi-classical model is small compared to data scatter .
2. In the data range covered, the zero net mass flow mode appears to be close to phase separator operation however the Gorter-Mellink constant is considerably lower than its bulk value for the large duct diameter range.
3. As He II is among the simplest non-Newtonian fluids, its porous media results appear to provide considerable insight improving our knowledge of classical Newtonian fluid behavior in porous media.

The latter point relates to the physical condition of zero net mass flow which eliminates gravitational direction dependence, e.g. Ref. (15). Further, it is noted that the normal fluid component of He II is Newtonian subject to He II constraints.

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